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淡江大學98學年度碩士班招生考試試題

系別：資訊工程學系
資訊工程學系資訊網路與通訊碩士班

科目：數　學(含離散數學、線性代數)

准帶項目請打「V」	
X	簡單型計算機

本試題共 1 頁，7 大題

1. True or false. (18 pts)

_____(a) Let A and B be two square matrices of the same size, then $det(A+B) = det(A)+det(B)$.

_____(b) If A is an $n\times n$ matrix and the system $AX = 0$ has nontrivial solutions, then A is not invertible.

_____(c) Every elementary matrix is invertible.

_____(d) If A is invertible then the reduced row-echelon form of A is an identity matrix.

_____(e) Let A, B, and C be matrices such that $AB = AC$, then $B = C$.

_____(f) The transformation $T: R^2\rightarrow R^3$ defined in the following is linear.

$w_1 = 2x_1 - 3x_2 + 5$
$w_2 = 5x_1 + 3x_2$
$w_3 = 4x_2 - 7$

2. Multiple choice (only one is the correct answer) (15 pts)

_____(a) How many even integers in { 100 ~ 999 } have no two digits the same?(i.e., every digit is different) ① 101 ~115 ② 116~130 ③ 131~145 ④ 146~160 ⑤ 161~175 ⑥ none of above.

_____(b) How many numbers must be selected from the set { 1, 3, 5, 7,..., 25} to guarantee that at least one pair of these numbers add up to 30? ① 13 ② 12 ③ 11 ④ 10 ⑤ 9 ⑥ 8 ⑦ none of above.

_____(c) How many permutations of 12345 are there that leave 3 in the third position but leave no other integer in its own position? ① 3 ② 4 ③ 5 ④ 6 ⑤ 7 ⑥ 8 ⑦ none of above.

_____(d) How many different strings can be made using all the letters in the word *GOOGOL*? ① 21～30 ② 31~35 ③ 36~40 ④ 41~45 ⑤46~50 ⑥ 51~60 ⑦ none of above.

_____(e) (from (d)) What if strings have to start with *O* or end with *O* ? ① 21～30 ② 31~35 ③ 36~40 ④ 41~45 ⑤46~50 ⑥ 51~60 ⑦ none of above.

3. Prove that $1^2 - 2^2 + 3^2 - \ldots + (-1)^{n-1} n^2 = (-1)^{n-1} n(n+1)/2$ by induction for $n \geq 1$. (15 pts)

4. Evaluate $U \cdot V$ and $\|UXV\|$ if $\|U\| = 2$, $\|V\| = 5$, $\|U+V\| = 1$, $\|U-V\| = 5$. (14 pts)

5. Determine whether $[(p\wedge q) \rightarrow r] \wedge s$ and $[p \rightarrow (q\wedge r)] \rightarrow s$ are equivalent. Explain. (12 pts)

6. Let $A = \begin{bmatrix} 1 & 0 \\ -5 & 2 \end{bmatrix}$. Find elementary matrices E_1 and E_2 such that $A = E_1E_2$. (14 pts)

7. Show that the "divide" relation on the set of positive integers is not an equivalence relation. (12 pts)