淡江大學 104 學年度碩士班招生考試試題 38

系別：資訊工程學系
資訊工程學系資訊網路與多媒體碩士班

科目：線性代數

考試日期：3 月 8 日(星期日) 第 3 節　　本試題共 5 大題，1 頁

1. **True/False** (15 pts)

_____(a). If matrix A is symmetric and matrix S is orthogonal, then matrix $S^{-1}AS$ must be symmetric.

_____(b). There exists a subspace V of R^3 such that $\dim(V) = \dim(V^{\perp})$ where $V^{\perp}$ is the orthogonal complement of V.

_____(c). A and A^t (the transpose of A) have the same eigenvalues where A is an $n\times n$ matrix.

_____(d). If a real matrix A has only the eigenvalues 1 and −1, then A must be orthogonal.

_____(e). Let A be a 3×3 matrix with characteristic equation $(\lambda+1)(\lambda -2)^2 = 0$. Then dimensions for the eigenspaces of A corresponding to the eigenvalues $\lambda = -1$ and $\lambda = 2$ are 1 and 2, respectively.

2. **Fill the blanks** (30 pts)

(a) Find the point of intersection of the planes $x + 2y - z = 1$, $x - 3y = -5$, and $2x + y + z = 0$ in R^3. **Ans①**

(b) Find the area of parallelogram whose vertices are (-1, 0), (0, 5), (1, -4), and (2, 1). **Ans②**

(c) Find the distance between the point (1, -4, -3) and the plane $2x - 3y + 6z = -1$. **Ans③**

(d) Find the rank and the nullity of the matrix $\boldsymbol{A}$ (given below): rank = **Ans④**; nullity = **Ans⑤**

(e) Find all eigenvalues of the matrix $\boldsymbol{B}$ (given below). **Ans ⑥**

$$\boldsymbol{A} = \begin{pmatrix} 1 & 1 & 2 & -1 \\ 0 & 1 & -4 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \boldsymbol{B} = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 1 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$

For the rest of problems, show detailed work to get full credits. Only half of the credits would be awarded at most when a problem is not solved by the specified method if there is any. 寫出詳細過程；如果沒有依照題目要求的作法將至多拿到一半的分數。

3. The vectors $\mathbf{u}_1 = (1, 2, 2)$, $\mathbf{u}_2 = (-1, 0, 2)$, $\mathbf{u}_3 = (0, 0, 1)$ form a basis for $\mathbf{R}^3$. (12+8 pts)

(a) Use these vectors in the Gram-Schmidt process to construct an orthonormal basis for $\mathbf{R}^3$.

(b) Find the distance from the point (0, 0, 1) to the subspace spanned by $\mathbf{u}_1$ & $\mathbf{u}_2$.

4. Consider the following: (12+8 pts)

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 0 \\ 3 & 0 & 3 \end{pmatrix} \qquad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \text{and} \quad b = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

(a). Use row operations to compute A^{-1}.

(b). Use Cramer's rule to solve for x without solving for y and z in the linear system $AX = b$.

5. Find the QR-factorization of the following matrix: $\begin{pmatrix} 4 & 25 & 0 \\ 0 & 0 & -2 \\ 3 & -25 & 0 \end{pmatrix}$ (15%)