淡江大學 95 學年度碩士班招生考試試題

89

系別：資訊工程學系　　　　科目：機　率　論

准帶項目請打「V」	
	簡單型計算機

本試題共 1 頁

1. Let A and B be events such that $P(A \cap B)=1/4$, $P(A^C)=1/3$, and $P(B)=1/2$, where A^C is the complement event of A. What is $P(A \cup B)$? (10%)

2. For events A and B, prove that $P(A \cap B) \geq P(A) + P(B) - 1$ (10%)

3. Choose independently two real numbers B and C at random from the interval $[-1, 1]$ with uniform distribution, and consider the quadratic equation $x^2 + Bx + C = 0$. Find the probability that the roots of this equation are both real. (10%)

4. Find integers n and m such that the following equation holds. (hint: consider a recursive binomial coefficient formula) (10%)

$$\binom{13}{5} + 2\binom{13}{6} + \binom{13}{7} = \binom{n}{m}$$

5. Let U, V be random numbers chosen independently from the interval $[0, 1]$ with uniform distribution. Find the cumulative distribution and density functions of variable $Y = U + V$. (10%)

6. Prove that for any three events A, B, and C, each having positive probability,

$$P(A \cap B \cap C) = P(A) \cdot P(B|A) \cdot P(C|A \cap B) \qquad (10\%)$$

7. Consider joint density function $f_{XY}(x, y) = (1 + x \cdot y)/8, 0 < x < 2, 0 < y < 2$ of random variables X and Y. Prove or disprove that X and Y are *independent*. (10%)

8. Let X and Y be random variables with positive variances. The *covariance* of X and Y is defined as $Cov(X, Y) = E((X-E(X))(Y-E(Y)))$, and the *correlation* of X and Y is defined as $\rho(X, Y) = Cov(X, Y)/\sqrt{\sigma^2(X)\sigma^2(Y)}$. Show that

   (a) $\sigma^2(X+Y) = \sigma^2(X) + \sigma^2(Y) + 2Cov(X, Y)$ (10%)

   (b) $0 \leq \sigma^2\left(\frac{X}{\sigma(X)} + \frac{Y}{\sigma(Y)}\right) = 2 \cdot (1 + \rho(X, Y))$ (hint: use (a)) (10%)

9. Recall that the moment generating function $g_X(t)$ for random variable X is defined as $g_X(t) = \sum_{k=0}^{\infty} \frac{\mu_k t^k}{k!} = E(e^{tX})$, where $\mu_k = E(X^k)$, provided the series converges. Suppose X is has range $[0, \infty)$ and density function $f_X(x) = \lambda e^{-\lambda x}$ (exponential density with parameter λ). Compute $g_X(t)$ and μ_k, for $k = 1, 2, 3$. (10%)