淡江大學 95 學年度碩士班招生考試試題

系別：統計學系　　　　科目：基礎數學(含微積分、線性代數)

准帶項目請打「V」	
	簡單型計算機

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1) (a) Let $f(x)=\begin{cases} x, & 0\le x\le 2 \\ 4-x, & 2<x\le 3 \end{cases}$. Is f continuous at $x=2$? Why?
Moreover, Is f differentiable at $x=2$? Why? (6%)

(b) Let $H(x)=\int_{a(x)}^{b(x)} f(t)\,dt$, where f is a continuous function and a,b are differentiable functions. Evaluate $\frac{dH(x)}{dx}$ at $x=2$. (4%)

2) Find the following limits: (12%)

(a) $\lim\limits_{x\to\infty}\frac{x-4}{x^2-9}$.

(b) $\lim\limits_{x\to 2}\frac{\sqrt{x^2+5}-3}{x^2-4}$.

(c) $\lim\limits_{x\to\infty}\left(x-\sqrt{x^2+x}\right)$.

3) Evaluate the following integrals: (8%)

(a) $\int_{-1}^{1} x^4\left(\sqrt{x^5+1}\right)^3 dx$.

(b) $\int_0^1 \frac{f(1-x)}{f(x)+f(1-x)}\,dx$, where f is a continuous function.

4) Let f be a continuous function on $[-a,a]$. (12%)

(a) If f is even, prove that $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$.

(b) If f is odd, prove that $\int_{-a}^{a} f(x)\,dx = 0$.

(c) If $\int_0^a f(x)\,dx = 3$, find $\int_0^a f(a-x)\,dx$.

5) Evaluate $\sum\limits_{x=-1}^{\infty}(x-1)x(x+1)\frac{t^{x-1}}{(x+1)!}$, for any constant $t>0$. (8%)

6) Let A be a square matrix such that $A^2-2A-3I=0$, where I is the identity matrix. Find the inverse of A. (6%)

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7) Find the Wronskian of the three functions e^x, xe^x, x^2e^x. Are these functions linearly independent? Why? (10%)

8) Let $A=\begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{bmatrix}$.

(a) Find a basis for the null space of A. (8%)

(b) Find a basis for the row space of A consisting entirely of row vectors from A. (8%)

9) (a) What is meant by an orthogonal matrix? (4%)

Let A be an $n \times n$ orthogonal matrix. Prove that

(b) $\|A\vec{x}\| = \|\vec{x}\|$ for all $\vec{x} \in R^n$. (6%)

(c) $A\vec{x} \cdot A\vec{y} = \vec{x} \cdot \vec{y}$ for all $\vec{x}, \vec{y} \in R^n$. (8%)

Where $\vec{u} \cdot \vec{v}$ is the Euclidean inner product of vectors $\vec{u}, \vec{v}\ (\in R^n)$ and $\|\cdot\|$ is the Euclidean norm in R^n.