95

# 淡江大學98學年度碩士班招生考試試題

系別：統計學系　　　　科目：基礎數學(含微積分、線性代數)

| 准帶項目請打「V」 | |
|---|---|
| | 簡單型計算機 |

本試題共 1 頁，9 大題

1. Show that for $x>0$, $\left(\int_x^\infty te^{-t^2/2}dt\right)^2 \le \int_x^\infty e^{-t^2/2}dt \times \int_x^\infty t^2e^{-t^2/2}dt$. (Hint: consider the inequality $\int_x^\infty (at+1)^2 e^{-t^2/2}dt \ge 0$, where $a$ is any real number.) (10%)

所有答案皆須附計算或證明過程否則不予計分

2. Evaluate the following limits

   a) $\lim_{h\to 0}\dfrac{(2+h)^5-2^5}{h}$. (6%)

   b) $\lim_{x\to 3}\dfrac{x^2-x-6}{x-3}$. (5%)

   c) $\lim_{n\to\infty} s_n$, where $s_n=\sum_{k=1}^n \dfrac{1}{k(k+1)}$. (8%)

3. Let the sequence $\{a_n\}_{n=1}^\infty$ be defined as $a_1=1$ and $a_{n+1}=\sqrt{2+a_n}$, $n\ge 1$. Evaluate $\lim_{n\to\infty} a_n$. (10%)

4. a) Prove that $\int_0^\pi x f(\sin x)\,dx = \dfrac{\pi}{2}\int_0^\pi f(\sin x)\,dx$, provided that the two integrals exist. (Hint : use change-of-variable to rewrite the integral by letting $u=\pi-x$.) (6%)

   b) Use 4a) to evaluate the integral $\int_0^\pi x(\cos x)^4 \sin x\,dx$. (7%)

5. Let $\{(x_1,y_1),(x_2,y_2),\cdots,(x_n,y_n)\}$ be a collection of the $n$ points on the $x$-$y$ plane. Suppose that $\sum_{i=1}^n x_i=0$ and $x_i\ne 0$, for some $i=1,2,\cdots,n$. Prove that the quantity $S(a,b)=\sum_{i=1}^n (y_i-ax_i-b)^2$ is minimized, when $a=\left(\sum_{i=1}^n x_i^2\right)^{-1}\sum_{i=1}^n x_iy_i$, $b=n^{-1}\sum_{i=1}^n y_i$. Justify your answer by second derivative test. (12%)

6. Find the value of $t$ for which the following system is consistent and solve the system for this value of $t$. (5%)

$$\begin{cases} x+y=1 \\ tx+y=t \\ (1+t)x+2y=3 \end{cases}$$

7. Let $C$ be an invertible matrix, show that $(C^{-1})'=(C')^{-1}$. (5%)

8. Let $A=\begin{pmatrix}\cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha\end{pmatrix}$, $B=(I-A)(I+A)^{-1}$ and $C=(I-B^3)(I+B^3)^{-1}$, where $I$ is the $2\times 2$ identity matrix.

   a) Show that $(I+A)^{-1}=(I+A^{-1})/\det(I+A)$ and hence show that $B$ is skew-symmetric (i.e. $B'=-B$). (Hint : $A$ is orthogonal, so $A'=A^{-1}$.) (8%)

   b) It is known that $(I+B^3)$ and $(I-B^3)$ are both invertible. Use 7) and 8a) to show that $C$ is orthogonal (i.e. $CC'=I$). (8%)

9. Let $A=\begin{bmatrix}4 & -3 \\ 1 & 0\end{bmatrix}$, use the fact $A^2=4A-3I_2$ and mathematical induction, to prove that (10%)

$$A^n=\frac{(3^n-1)}{2}A+\frac{(3-3^n)}{2}I_2,\ \text{for } n\ge 1,$$

where $I_2$ is the $2\times 2$ identity matrix.