

90

淡江大學 99 學年度碩士班招生考試試題

系別：統計學系

科目：基礎數學(含微積分、線性代數)

准帶項目請打「V」	
✓	計算機
本試題共 1 頁，9 大題	

下列所有計算或證明題，須附計算或證明過程，否則不與計分。

1. Evaluate the following limits:

a) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 + y^4}{x^2 - y^2}$ (5%)

c) $\lim_{n \rightarrow \infty} \frac{(1/2n)^p + (2/2n)^p + \dots + (2n/2n)^p}{[(1/2) + (1/2n)]^p + [(1/2) + (2/2n)]^p + \dots + [(1/2) + (n/2n)]^p}$ ($p > 0$) (10%)

2. For $x > 0$, show that $e^x > 1 + \ln(1+x)$. (10%)

3. Suppose $\frac{1}{x^4 + 1} = \frac{1}{2\sqrt{2}} \left(\frac{x + \sqrt{2}}{x^2 + \sqrt{2}x + 1} - \frac{x - \sqrt{2}}{x^2 - \sqrt{2}x + 1} \right)$, find $\int \frac{dx}{x^4 + 1} = ?$ (10%)

4. Set $f(x) = \frac{1}{x}$, $a = -1$, $b = 1$, verify that there is no number c for which $f'(c) = \frac{f(b) - f(a)}{b - a}$ and explain how this doesn't violate (違反) the Mean Value theorem. (10%)

5. Find the approximate value of $\int_b^1 \frac{\sin x}{x} dx$ up to six decimal place (小數點第 6 位). (10%)

6. Mark each of the following True or False (是非題) (15%; each 3%)

- a) Cramer's rule, in theory, can be used to find all solutions of any square linear system.
- b) Every vector space has at least two distinct (不同的) subspaces.
- c) Let A be a square matrix. If A^3 is invertible, then A^2 is invertible.
- d) Every symmetric idempotent matrix is the projection matrix for its row space.
- e) If two rows and also two columns of a square matrix are interchanged (互換), the determinant changes sign.

7. Let I be an $m \times m$ identity matrix, and let A be an $m \times m$ nonsingular matrix with eigenvalues $\lambda_1, \dots, \lambda_m$ and corresponding eigenvectors x_1, \dots, x_m . If $I + A$ is nonsingular, find the eigenvalues and eigenvectors of

a) $(I + A)^{-1}$ b) $A + A^{-1}$. (10%; each 5%)

8. Find the QR decomposition of $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. (10%)

9. Let A be an $m \times n$ matrix and B be an $n \times p$ matrix. Show that

- a) the column space of AB is a subspace of the column space of A . (5%)
- b) if $\text{rank}(AB) = \text{rank}(A)$, then the column space of AB is equal to the column space of A . (5%)