

淡江大學 107 學年度碩士班招生考試試題

系別：資訊工程學系聯招

科目：線性代數

30-1

考試日期：3月11日(星期日) 第2節

本試題共5大題，1頁

1. True or False (20%)

- (a). For every square matrix A , $\det(A^T) = \det(A)$.
- (b). The vector equation of a plane can be determined from any point lying in the plane and a nonzero vector parallel to the plane.
- (c). If A and B are square matrices of the same size and A is invertible, then $\det(A^{-1}BA) = \det(B)$.
- (d). If $\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{w}$, then $\mathbf{u} = \mathbf{w}$.
- (e). If A is a square matrix having the eigenvalue $\lambda = 0$, then A is invertible.
- (f). The points $(1, 2, 3)$, $(2, 0, -1)$, $(4, 1, 1)$, and $(-2, 0, -1)$ lie in the same plane.
- (g). Let $A = \begin{bmatrix} 1 & 0 \\ -5 & 2 \end{bmatrix}$, then there exist elementary matrices E_1 and E_2 such that $A = E_1E_2$.
- (h). *Gram-Schmidt process* can be performed on any nonempty set of linearly independent vectors in R^n .
- (i). If S_1 and S_2 are two linearly dependent sets of vectors, then so is the union $S_1 \cup S_2$.
- (j). If X_0 is a least squares solution of the linear system $AX = b$ and $AX_0 = b$, then b must lie in the column space of A .

For Problems 2- 5, show the detailed work to get full credits.

2. Find the (a) scalar projection and (b) vector projection of \mathbf{v} onto \mathbf{u} when $\mathbf{u} = (2, 3)$ and $\mathbf{v} = (4, 1)$. (20%)

3. Use **Cramer's rule** to solve
- $$\begin{aligned} 2x + 3y - z &= 1 \\ 4x + y - 3z &= 11 \\ 3x - 2y + 5z &= 21 \end{aligned} \quad (20\%)$$

4. Find eigenvalues and eigenvectors of $\begin{bmatrix} 2 & -4 \\ -1 & -1 \end{bmatrix}$. (20%)

5. Given a linear transform $T: R^2 \rightarrow R^2$, find the standard matrix T where (20%)

$$T \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \text{and} \quad T \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}.$$